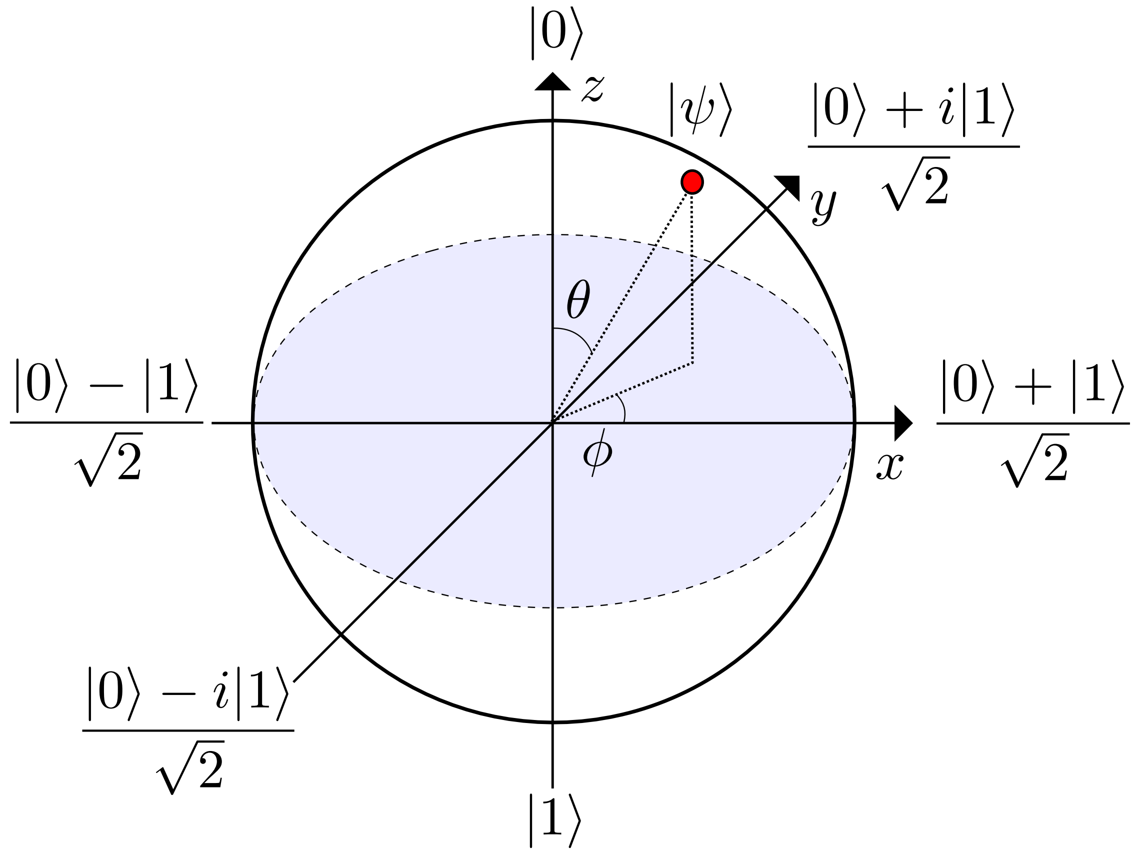
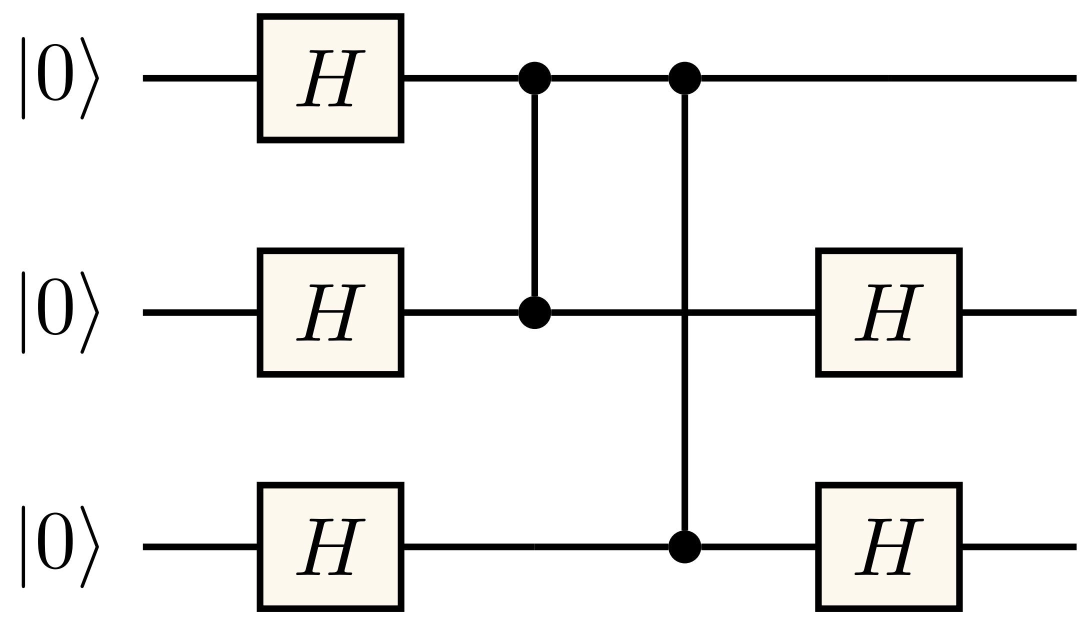
WACQT | Wallenberg Centre for Quantum Technology



Quantum states, qubits, logic gates, and algorithms



NordIQuEst

ENCCS
EuroCC National Competence Centre Sweden

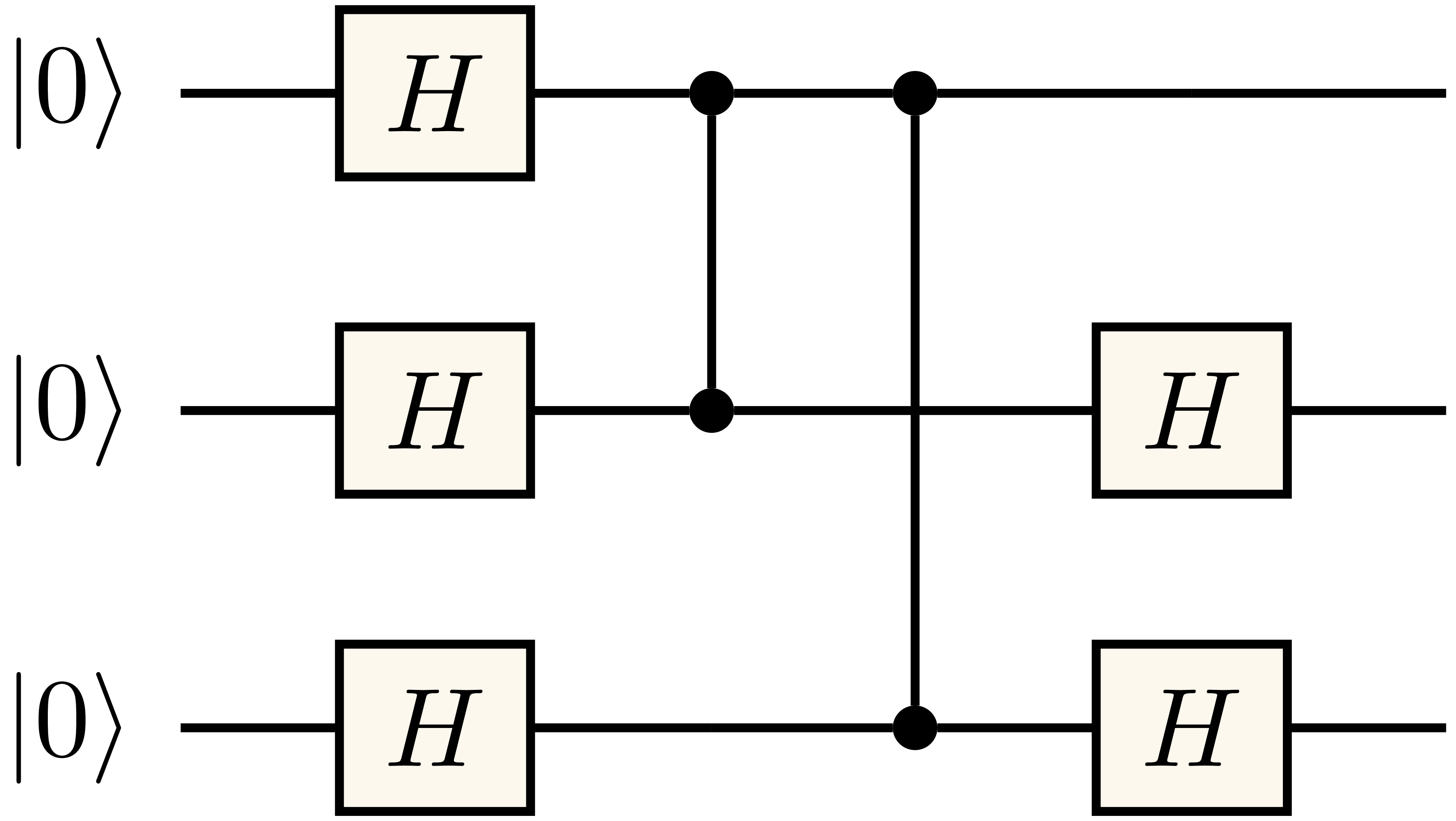
Anton Frisk Kockum

Senior Researcher, WACQT

Outline

- Components of a quantum circuit
- Quantum bits
- Single-qubit gates
- Multi-qubit gates
- Universal gate sets
- The Solovay-Kitaev theorem
- Quantum algorithms and compilation
- Summary

Components of a quantum circuit



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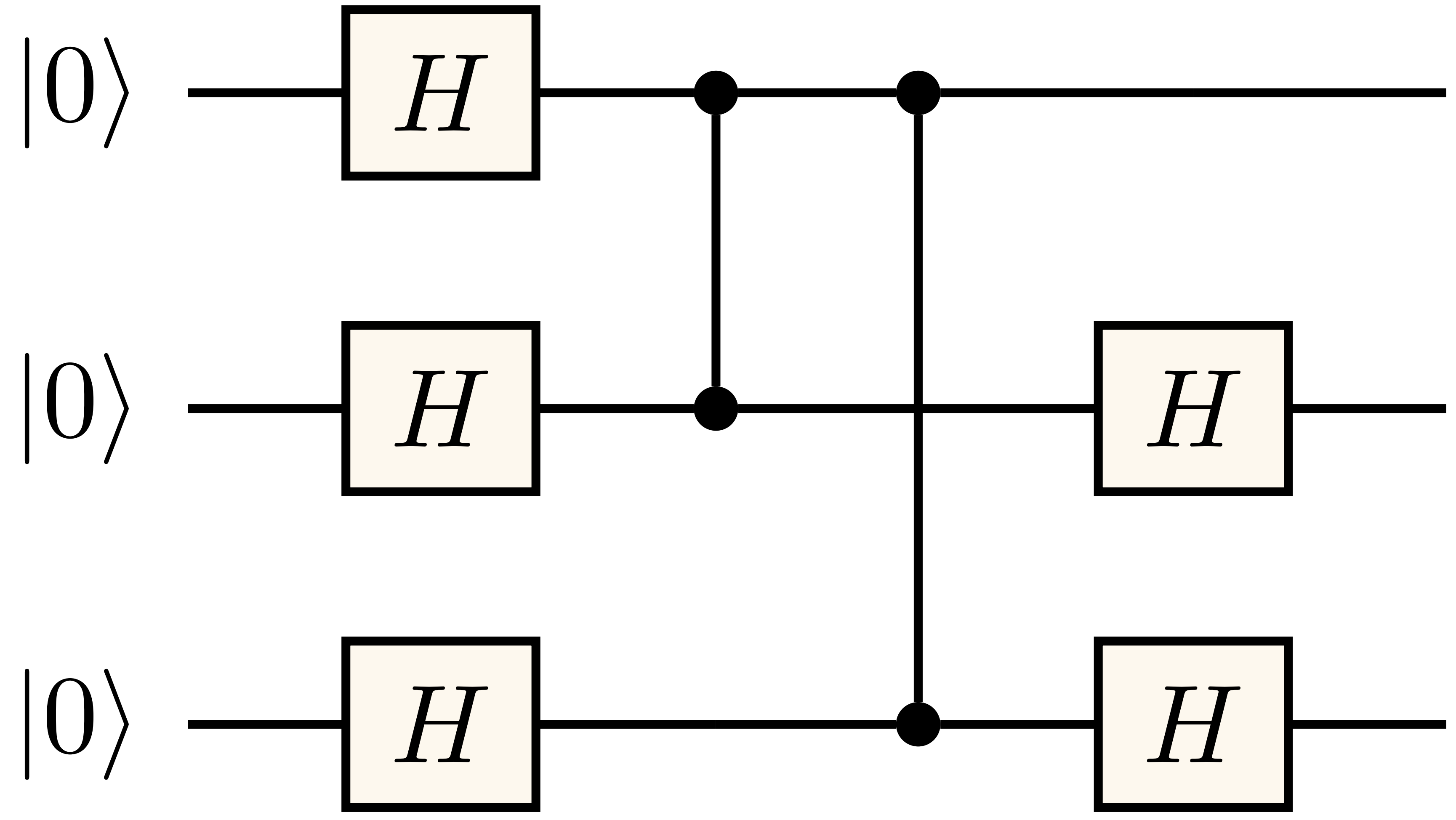
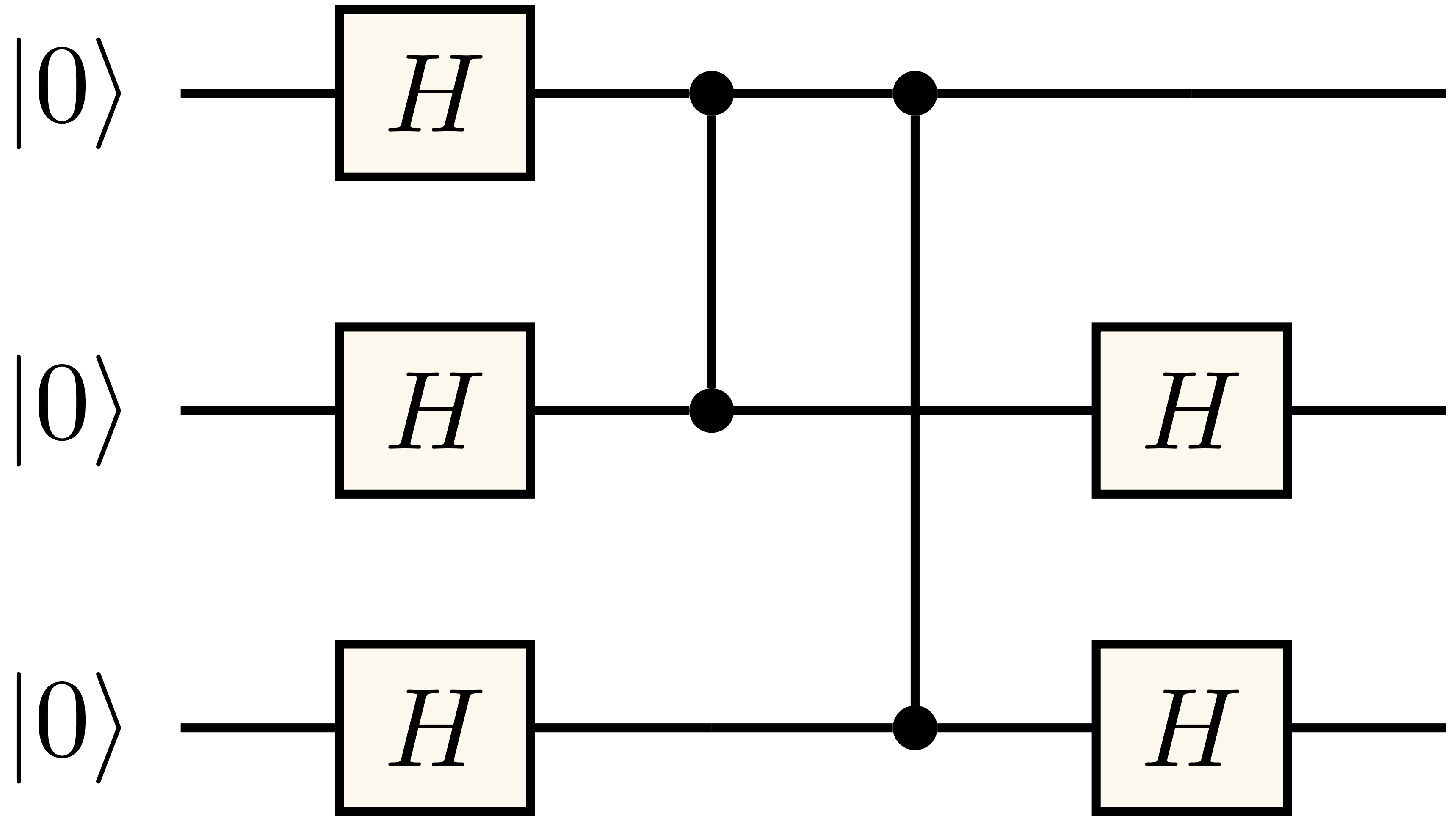


- Represent data: quantum bits

Components of a quantum circuit



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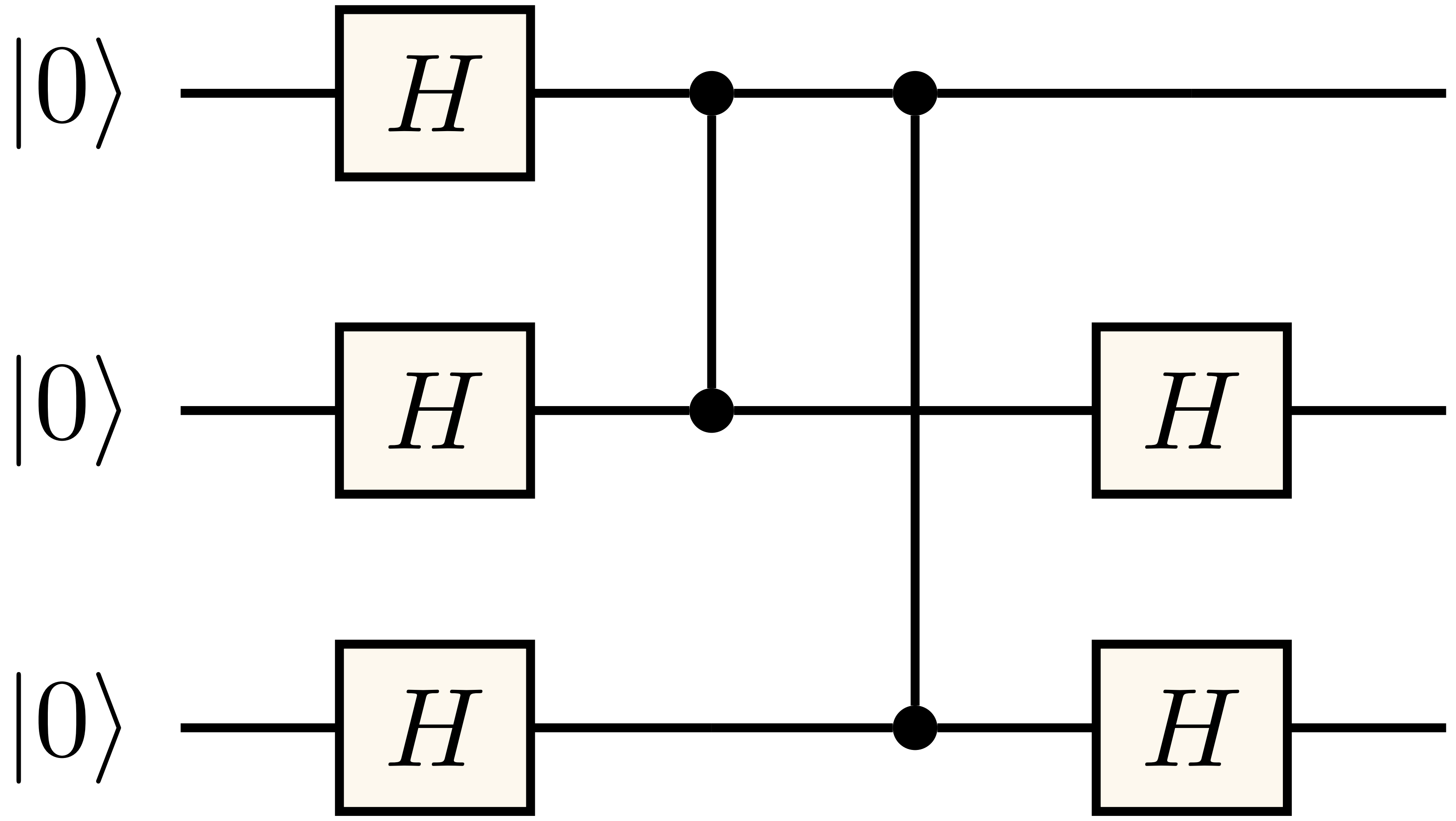

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- Initialize the computation: state preparation
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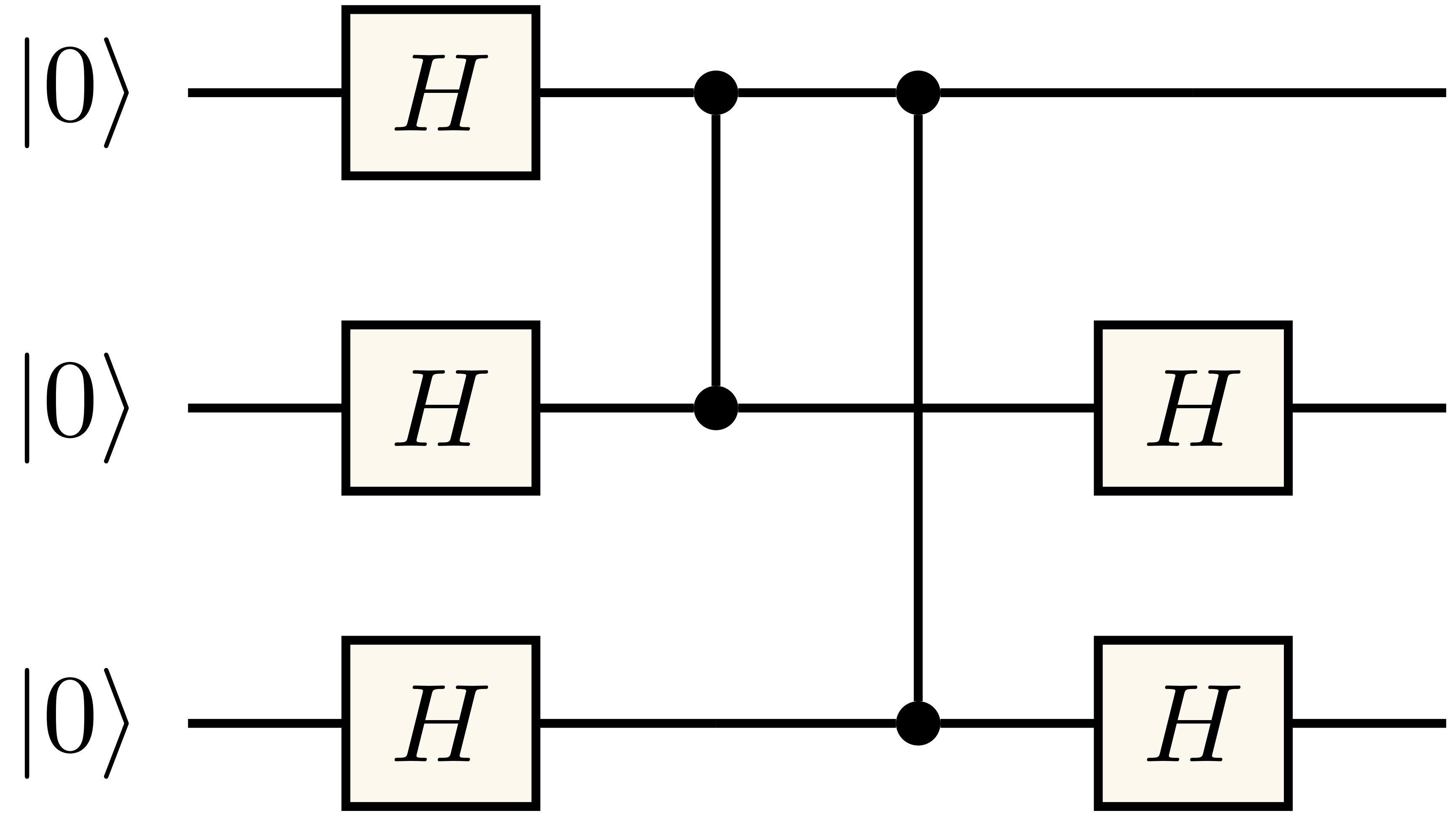

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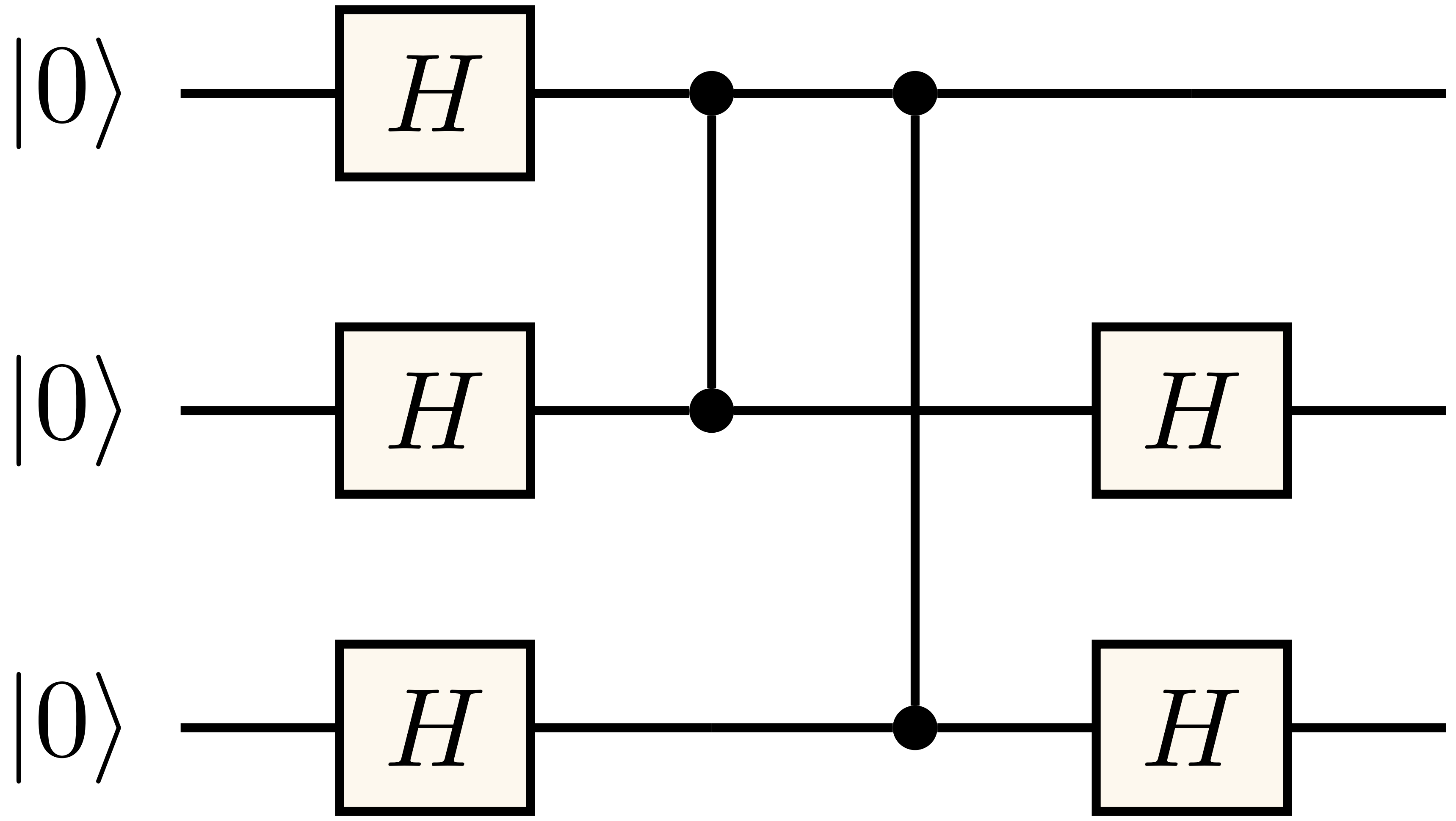

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$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \alpha\begin{pmatrix}1\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}\alpha\\\beta\end{pmatrix}$$

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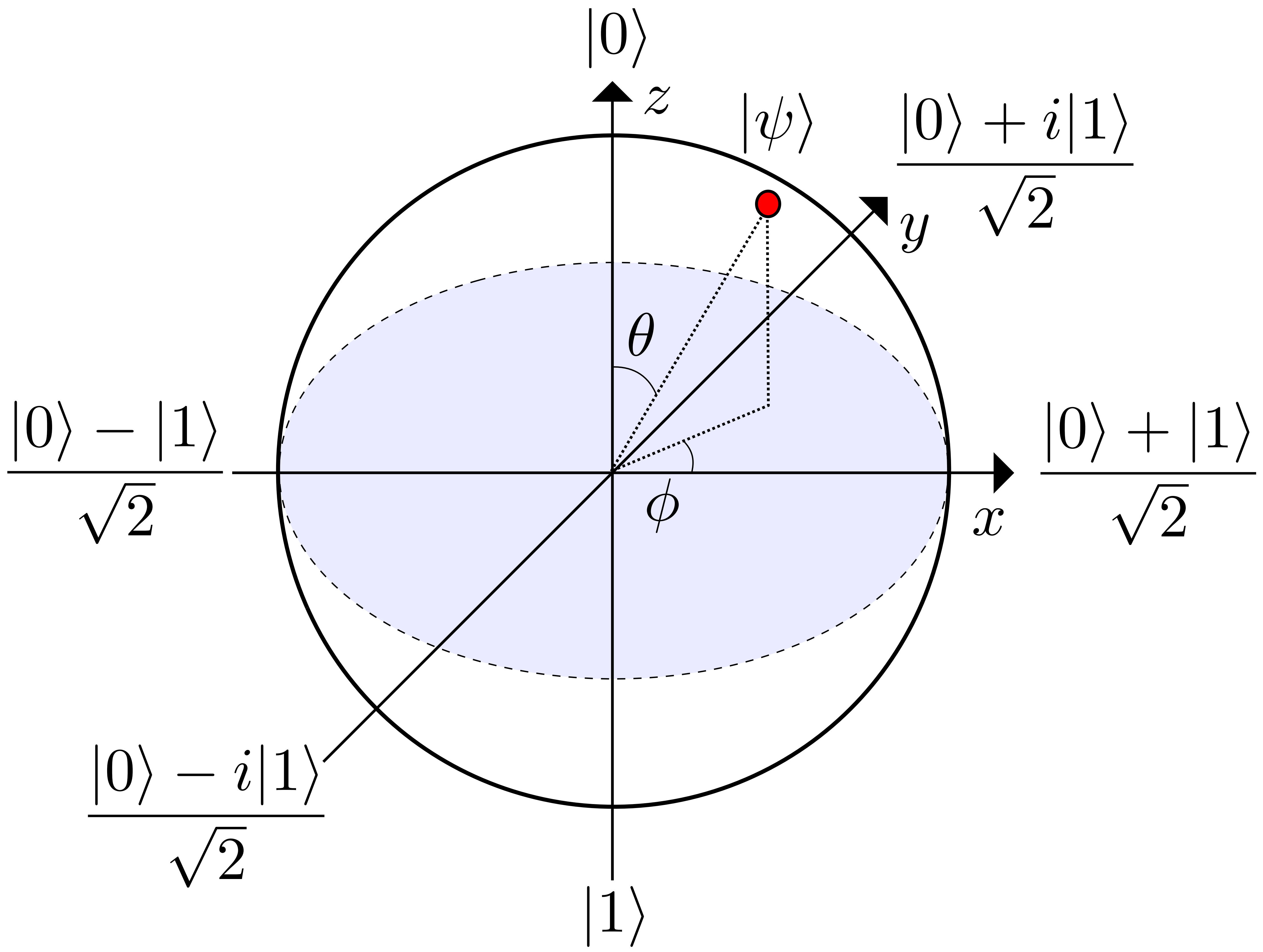

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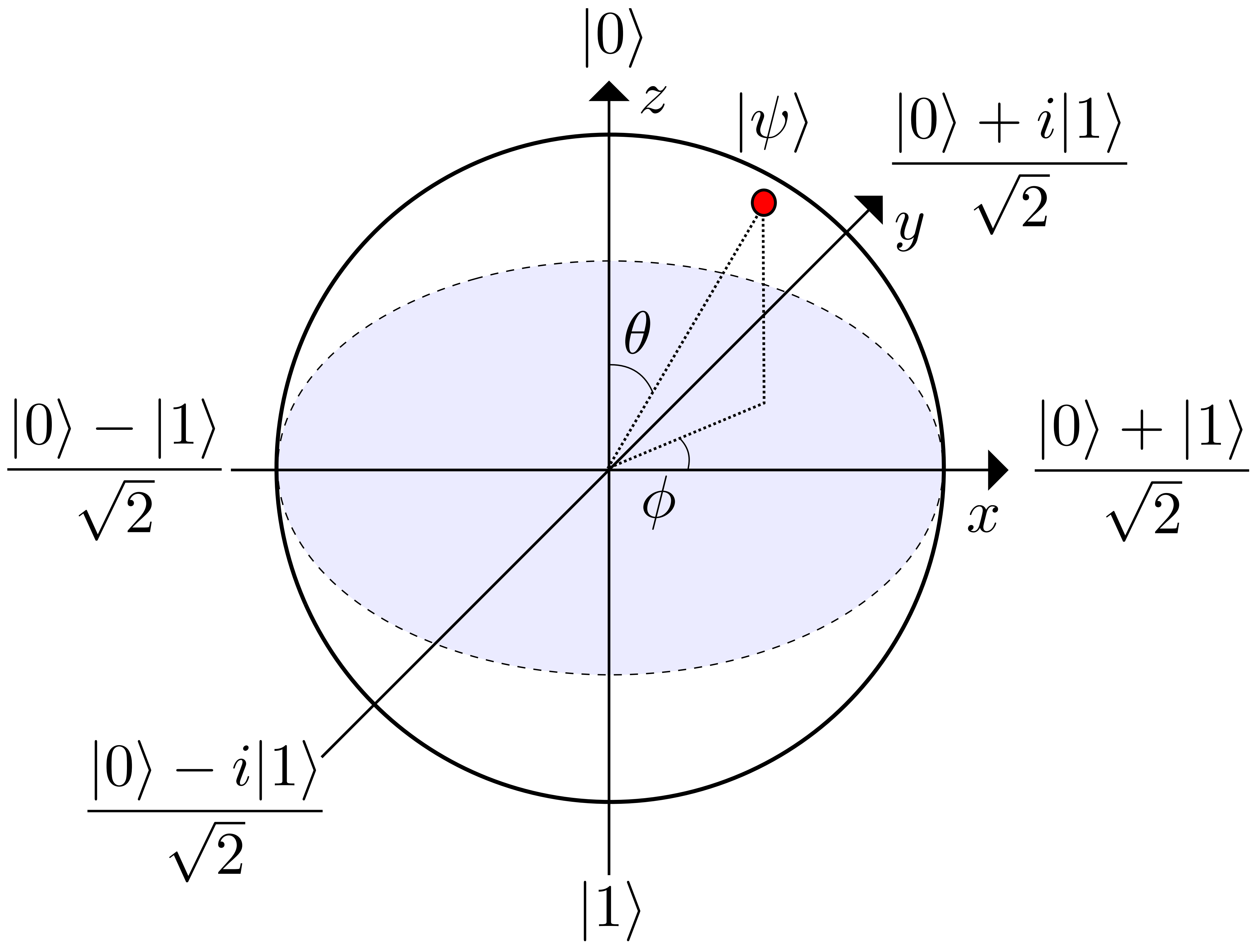
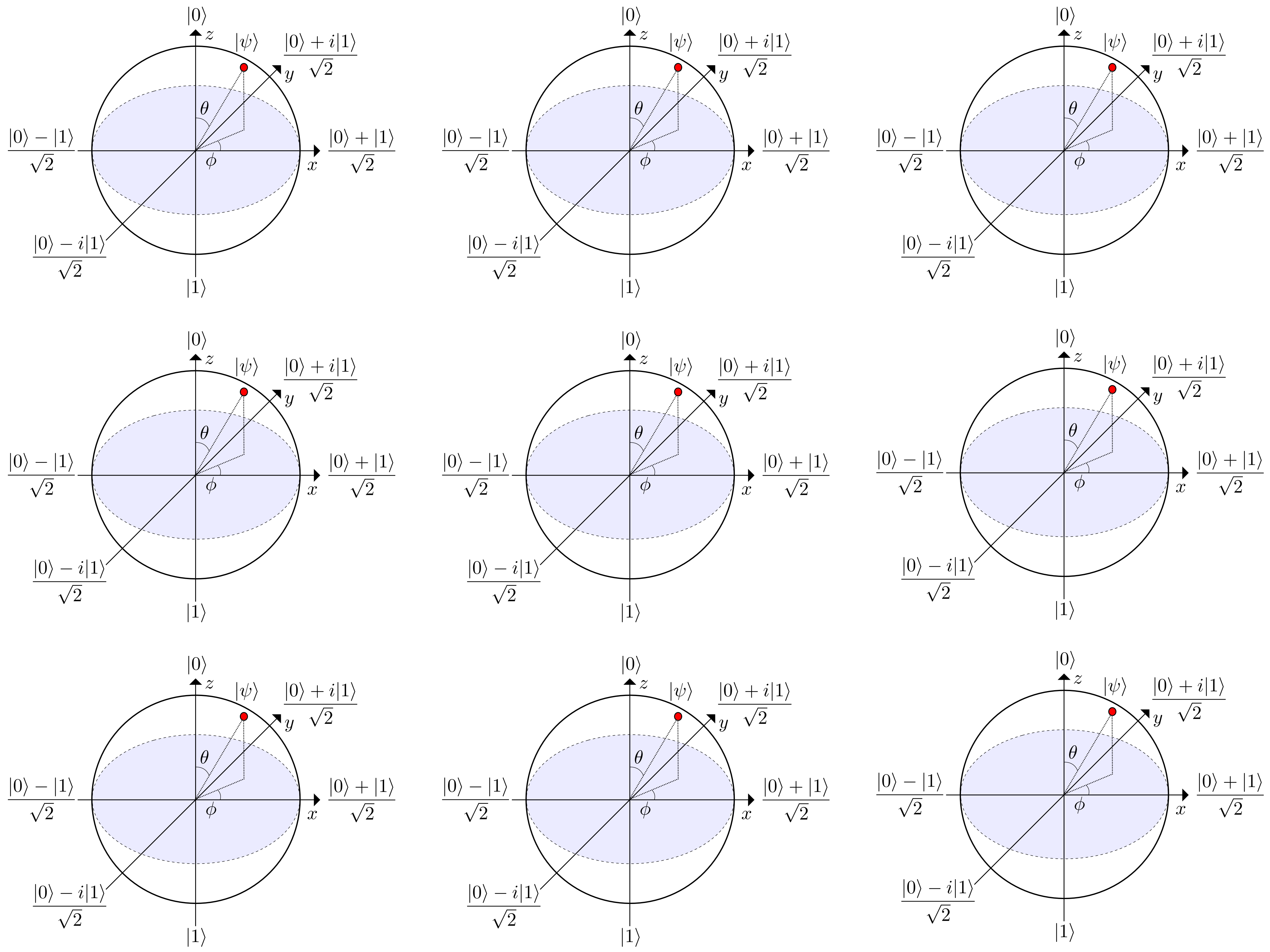
$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

Measurements give either 0 or 1 with probabilities $|\alpha|^2$ and $|\beta|^2$



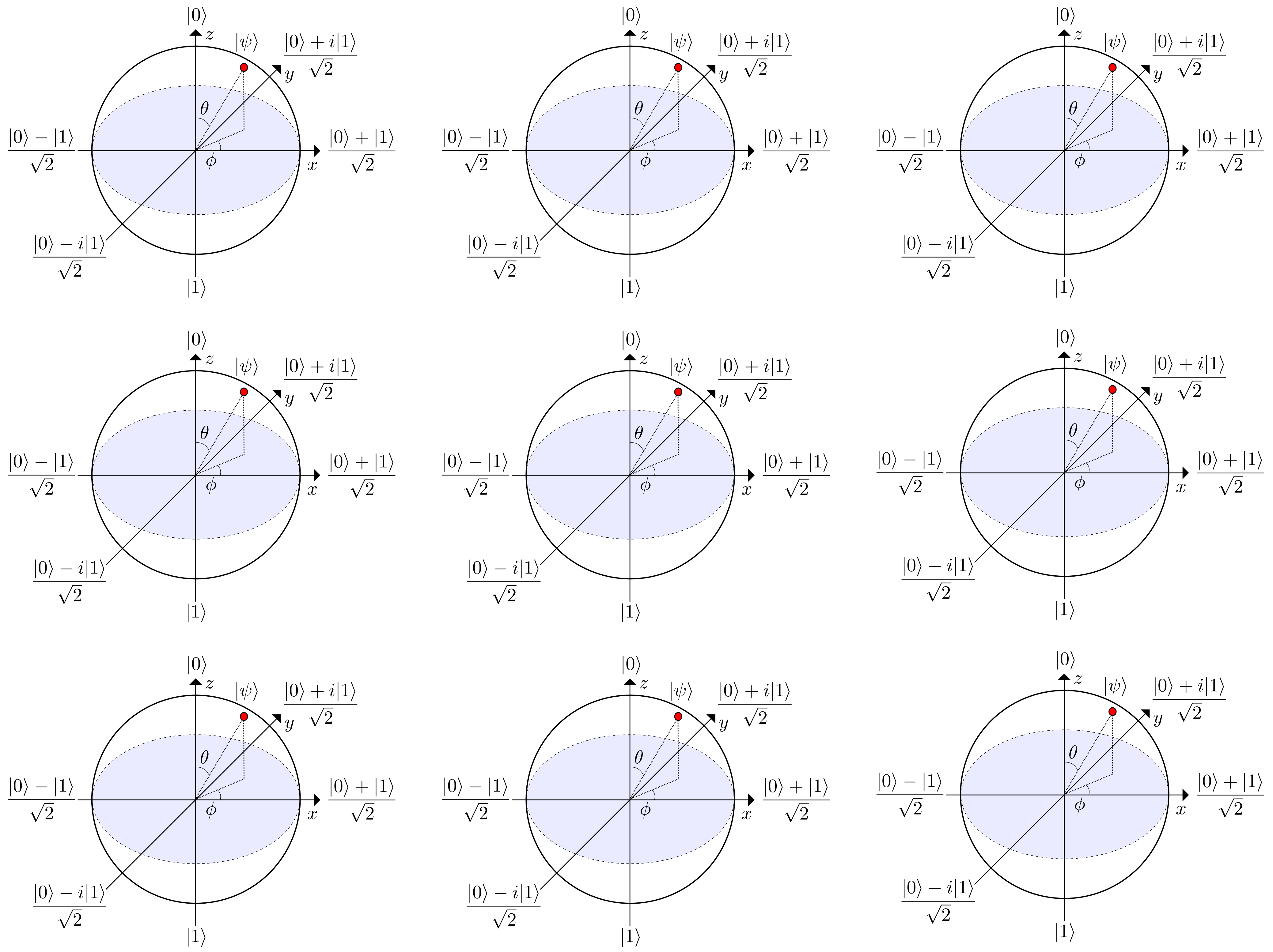
Multi-qubit states



Multi-qubit states

N qubits can be in a superposition of 2^N states

$$|000\ldots00\rangle, |100\ldots00\rangle, |010\ldots00\rangle, \ldots, |111\ldots10\rangle, |111\ldots11\rangle$$



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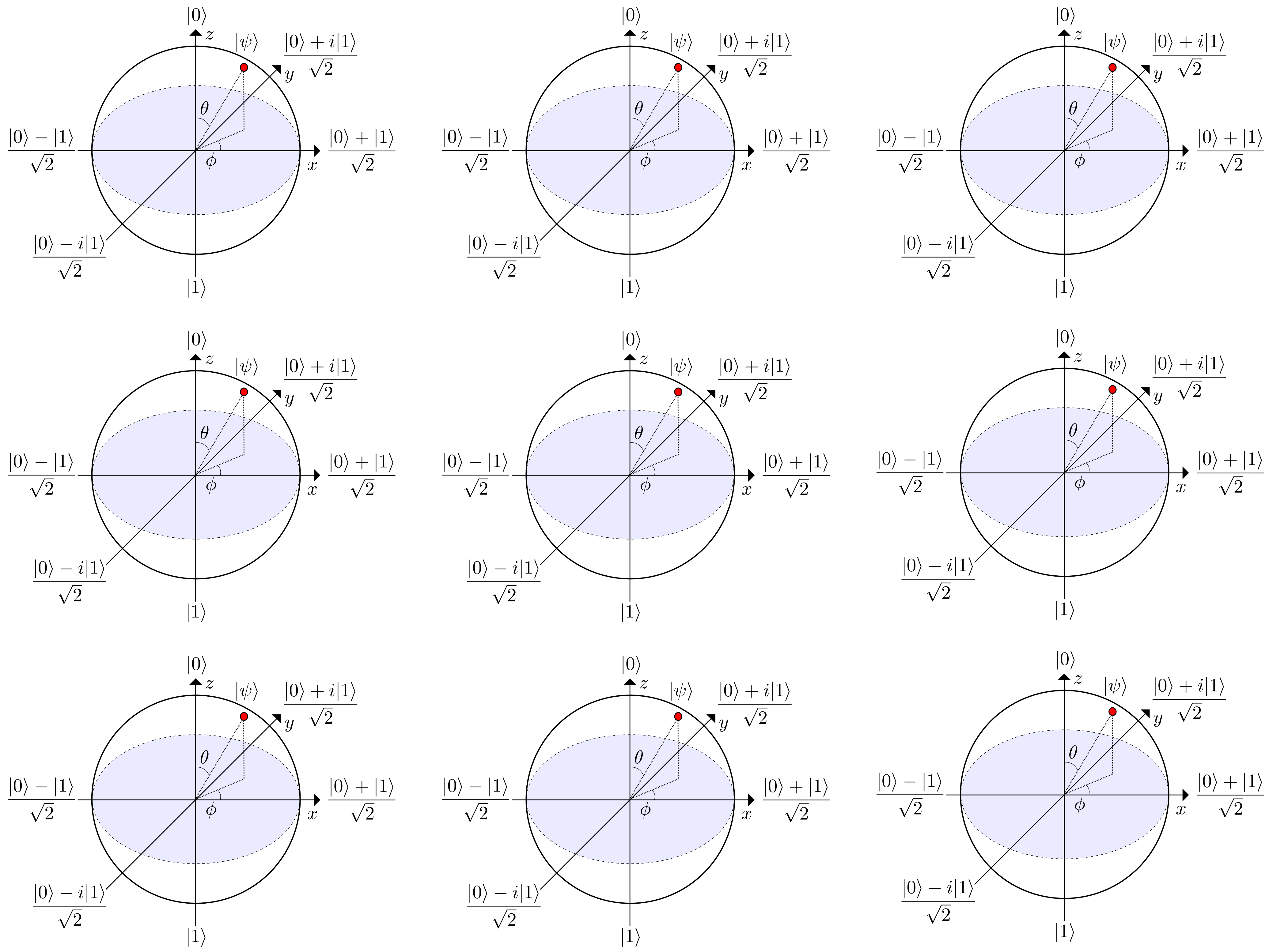

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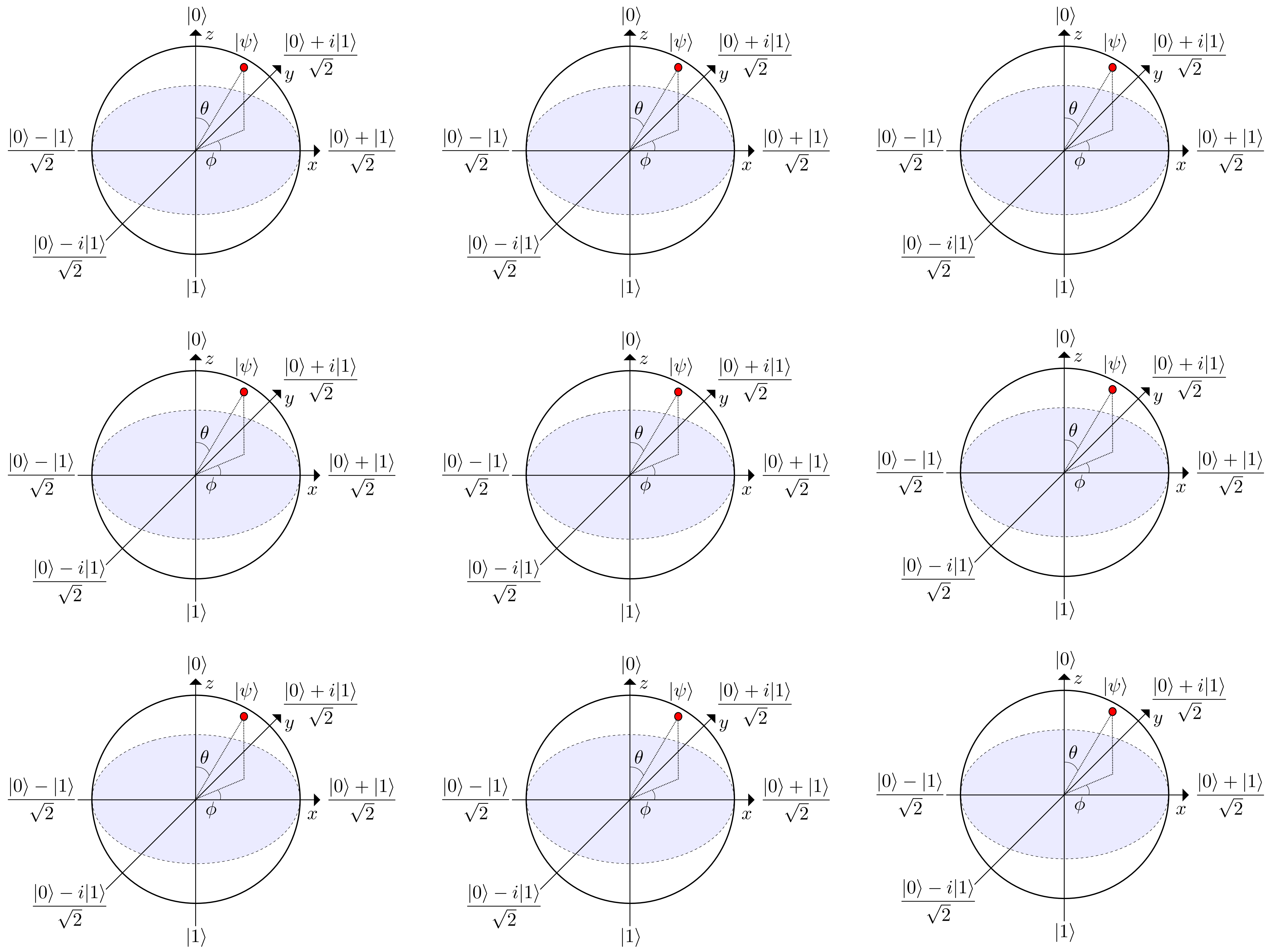
Storing all the information about a quantum state can require $\gg N$ classical bits



Single-qubit gates

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Operations changing the state of a qubit must preserve the norm

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They are 2x2 unitary matrices, e.g., the Pauli matrices

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

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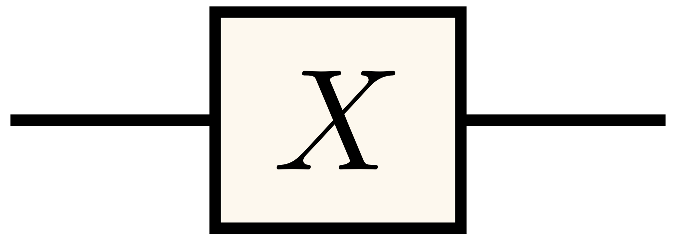

is the quantum NOT gate

$$X(\alpha|0\rangle + \beta|1\rangle) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix} = \beta|0\rangle + \alpha|1\rangle$$

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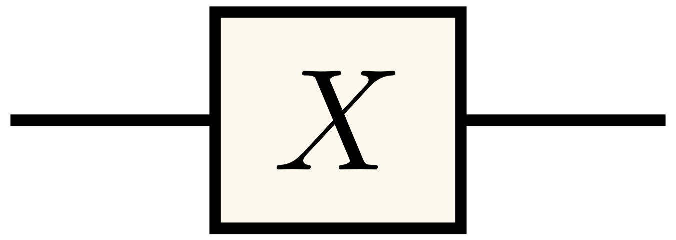

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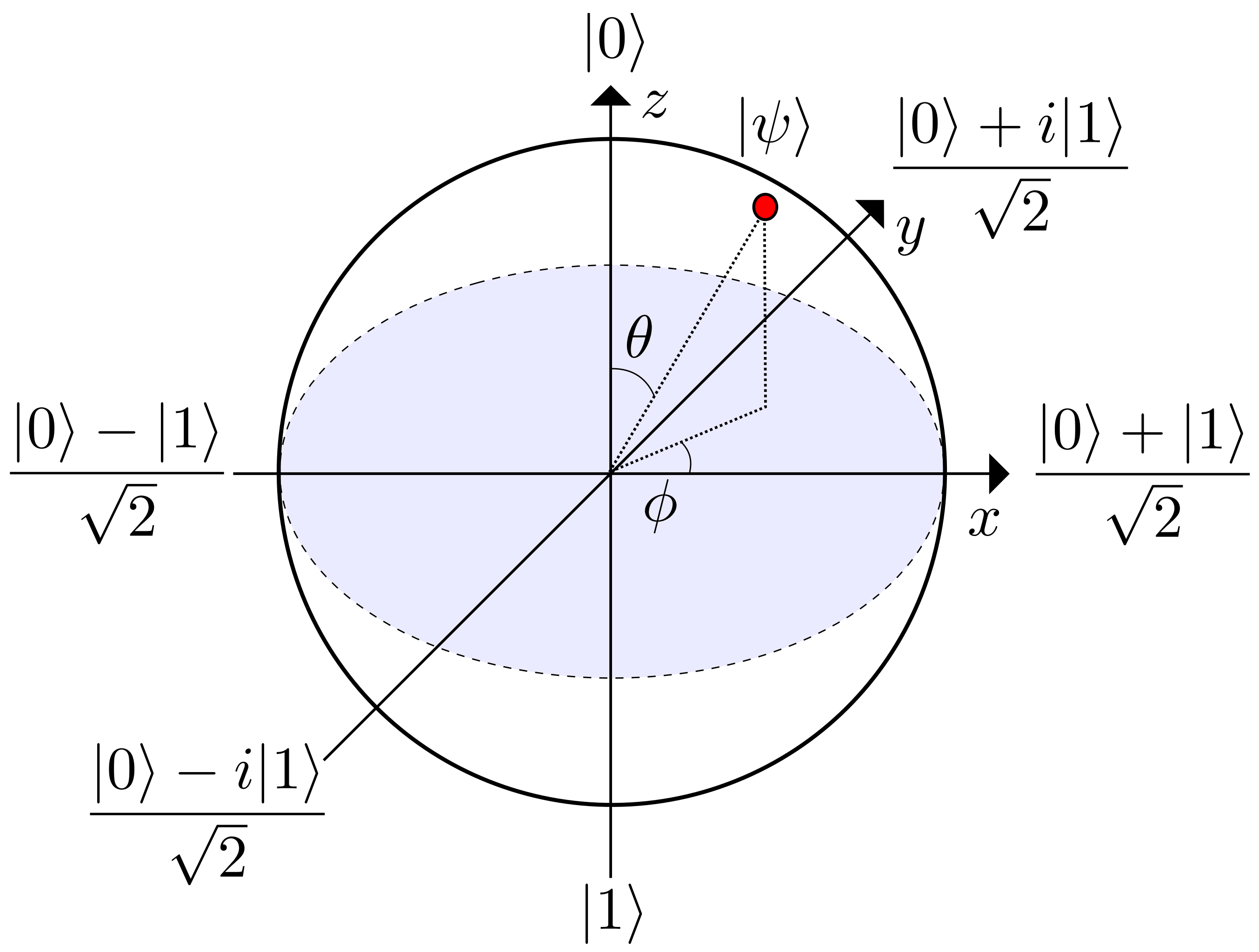
Rotations around different axes of the Bloch sphere

$$\begin{aligned} R_x(\theta) &= \exp(-i\theta X/2) \\ &= \cos(\theta/2)I - i\sin(\theta/2)X \\ &= \begin{pmatrix} \cos(\theta/2) & -i\sin(\theta/2) \\ -i\sin(\theta/2) & \cos(\theta/2) \end{pmatrix} \end{aligned}$$



More single-qubit gates

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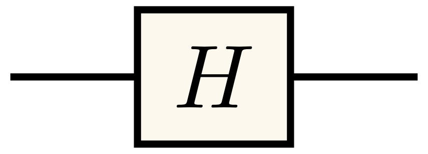

The Hadamard gate

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{X + Z}{\sqrt{2}}$$

More single-qubit gates

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$-\boxed{T}-$ The T gate ($\pi/8$ gate) $T = \begin{pmatrix}1 & 0 \\ 0 & \exp(i\pi/4)\end{pmatrix} = \exp(i\pi/8)\begin{pmatrix}\exp(-i\pi/8) & 0 \\ 0 & \exp(i\pi/8)\end{pmatrix}$

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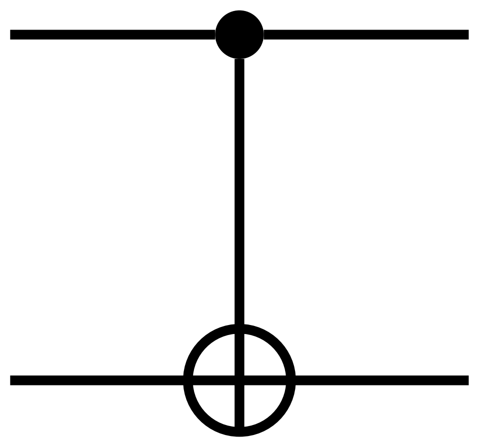
$-\boxed{S}-$ **The phase (or S, or P) gate** $$S = \begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix} = T^2$$

Two-qubit gates

Two-qubit gates

Controlled-NOT $\mathrm{CNOT} = \begin{array}{c c} \begin{array}{cccc} |00\rangle & |01\rangle & |10\rangle & |11\rangle \end{array} & \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} & \begin{array}{c} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array} \end{array}$



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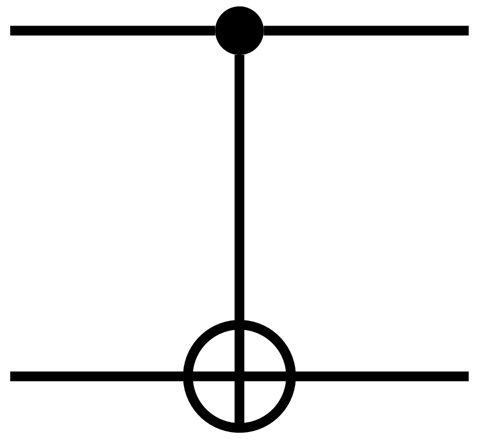

Controlled-Z

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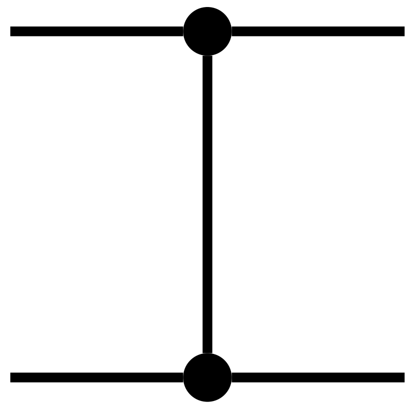

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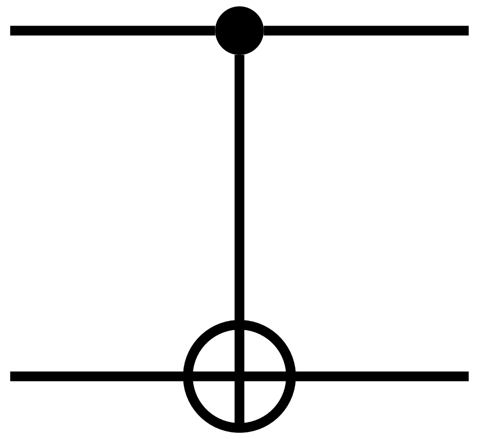

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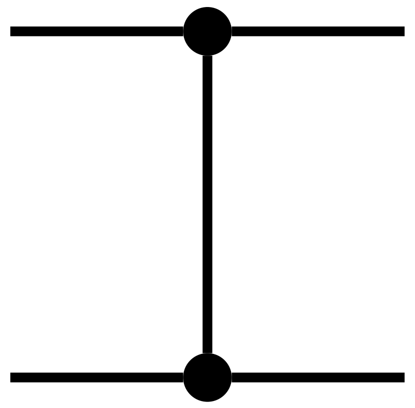

Controlled unitary

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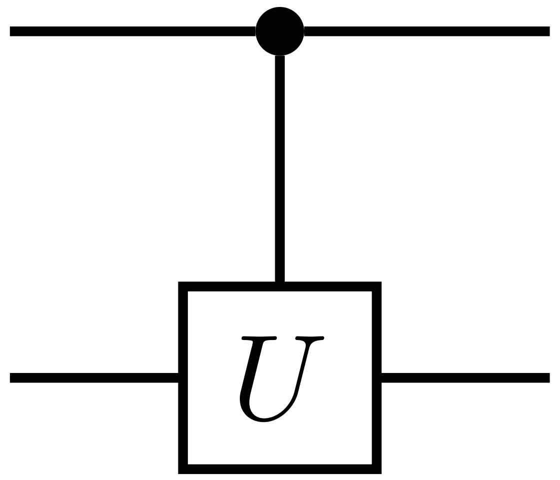

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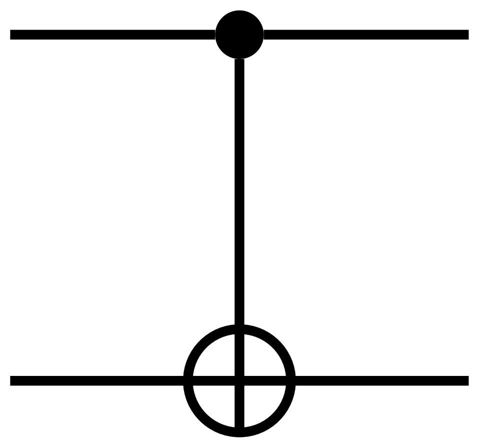

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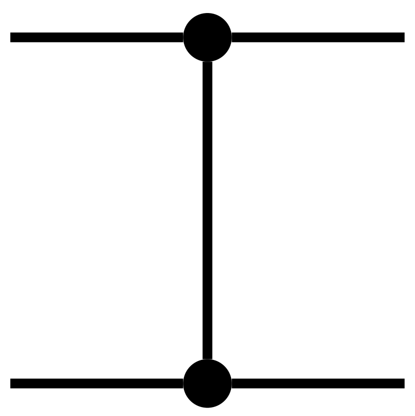
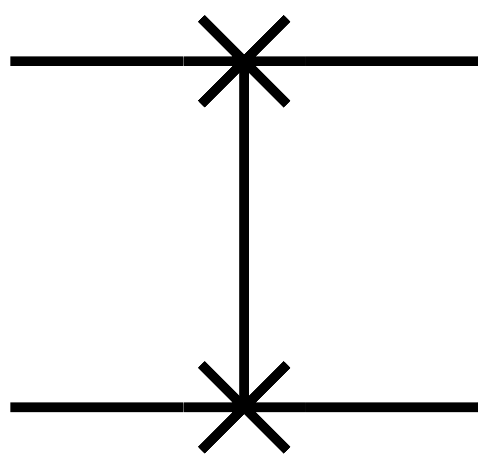


SWAP

$$\mathrm{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



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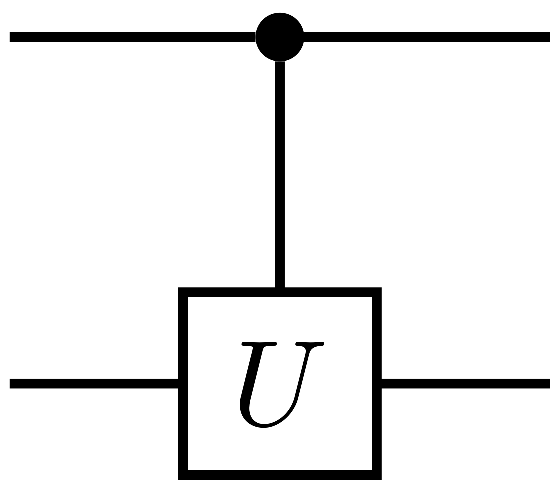
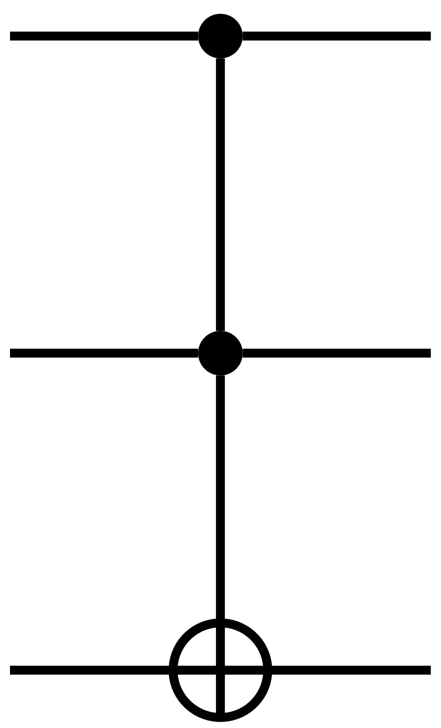


Three-qubit gates

Three-qubit gates

Controlled-controlled-NOT

$$\text{Toffoli} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$



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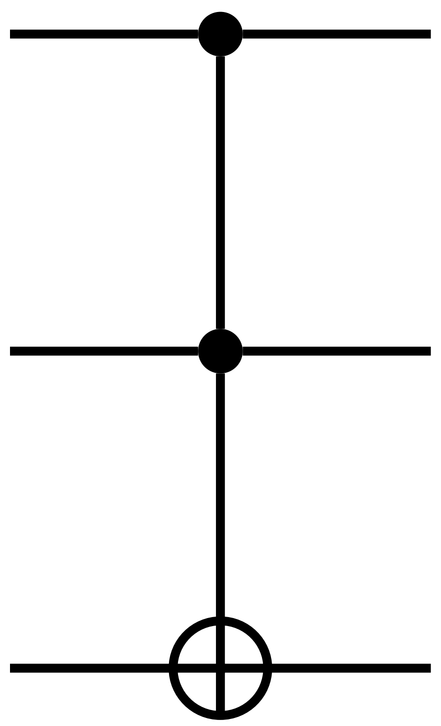
$$



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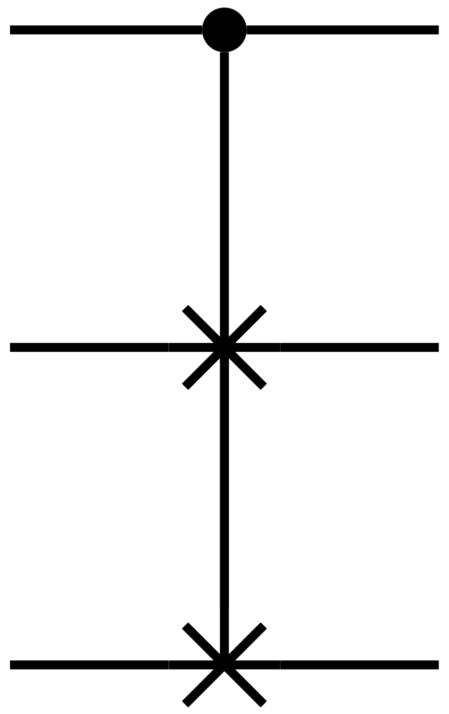

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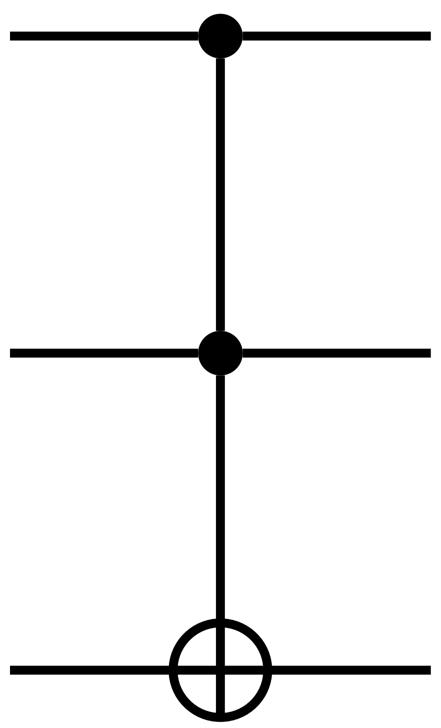

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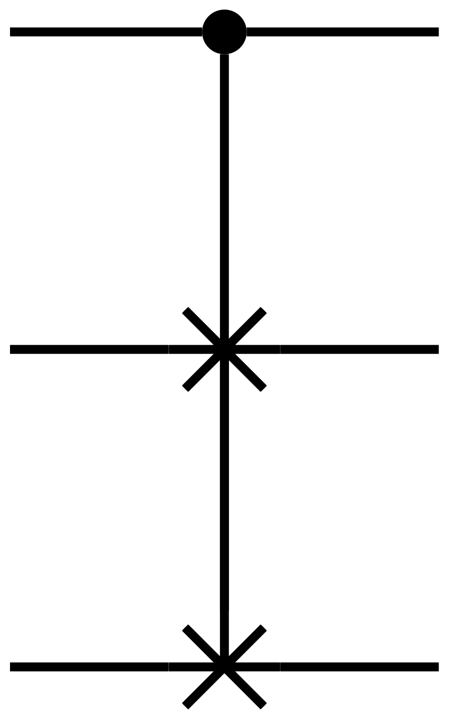

Multi-qubit gates can be decomposed into sequences of single- and two-qubit gates

Universal gate sets

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Classical computing

A gate set is universal if it enables expressing any Boolean function on any number of bits

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What are requirements for a universal set of quantum gates?

Universal quantum gate sets

Failure modes

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- Inability to create superposition states
  {X, CNOT}

Universal quantum gate sets

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- Inability to create superposition states
  {X, CNOT}
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- The Gottesman-Knill theorem
  {H, CNOT, S} still not enough!

Universal quantum gate sets

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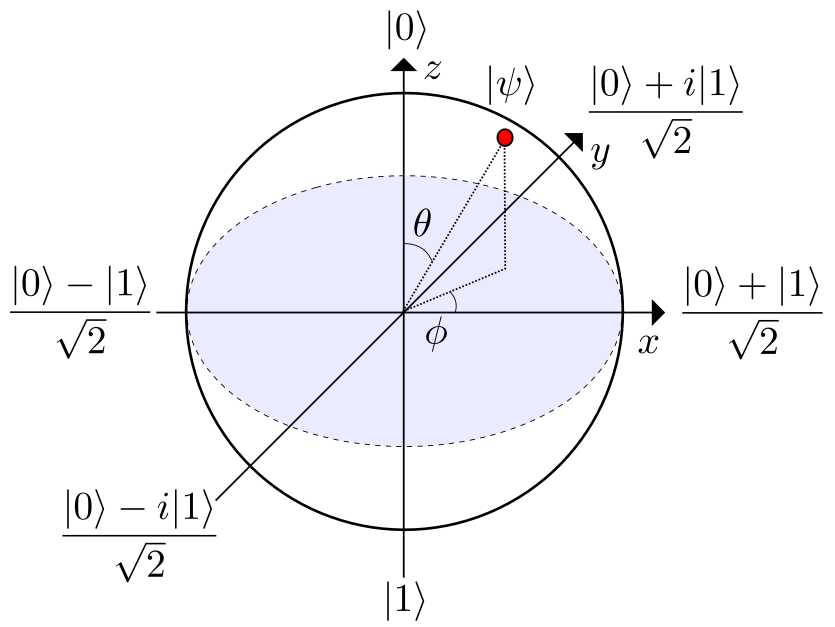

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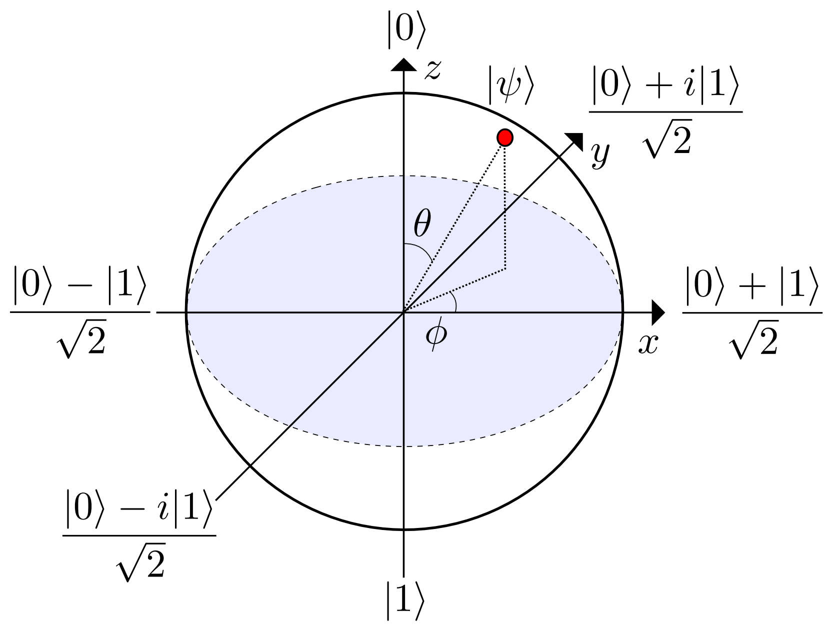

Universal gate sets

- Almost anything else than
  H in {H, CNOT, S}
- Almost any two-qubit gate
  on its own
- In practice: many single-qubit gates
  + one or two two-qubit gates

Quantum versus classical computing

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For the quantum computer to be faster, one thing to worry about is whether the universal gate set can represent the desired algorithm with enough precision without requiring too long circuits

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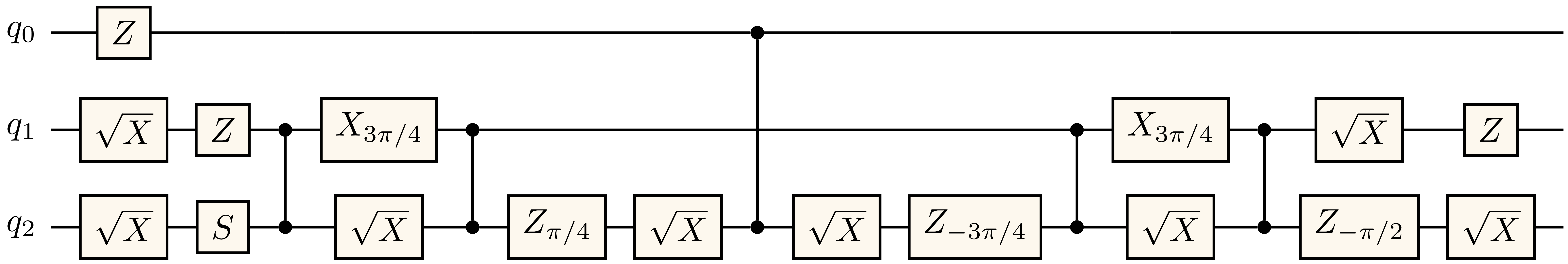
Precision is thus not a problem in practice for available universal gate sets

Quantum algorithms and compilation



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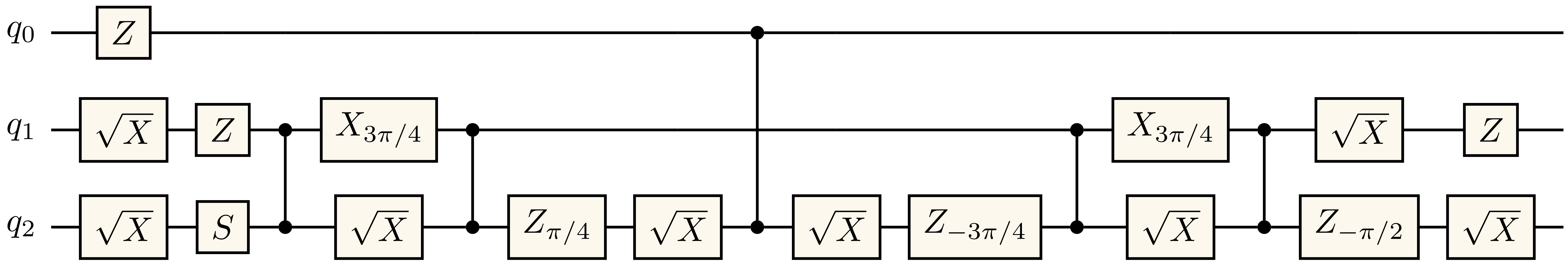
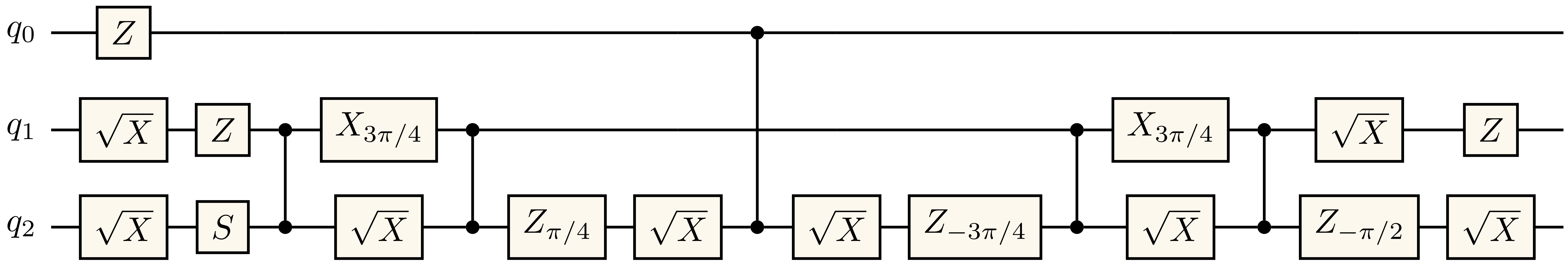


Quantum algorithms are sequences of gates acting on quantum states

Quantum algorithms and compilation



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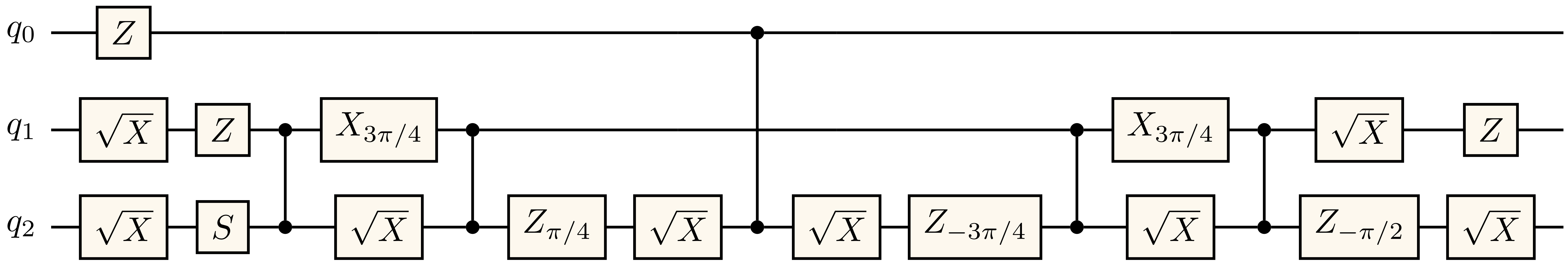
<u>Compilation steps</u>

Quantum algorithms and compilation



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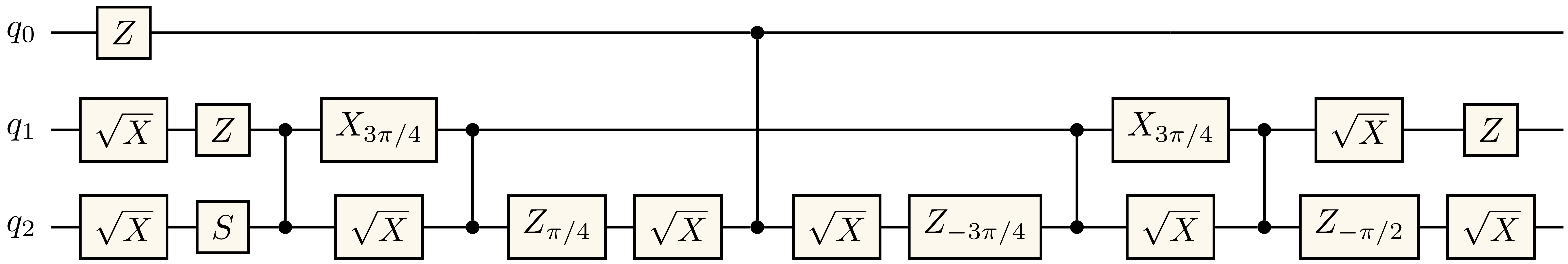

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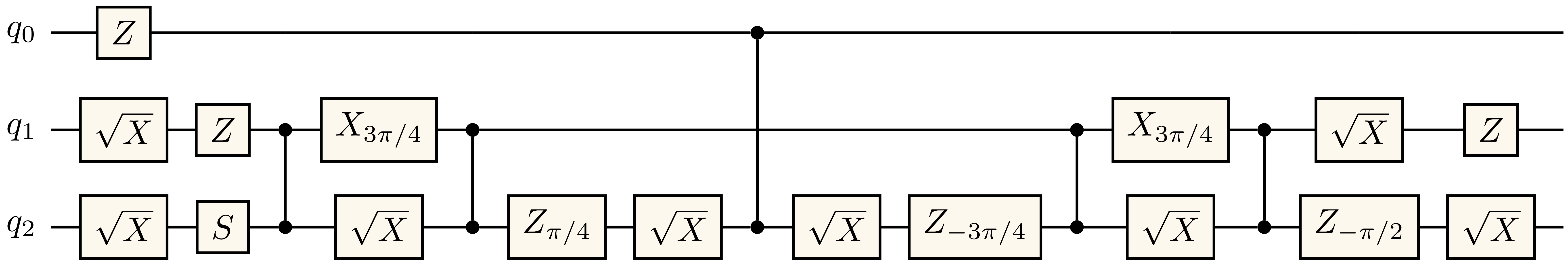

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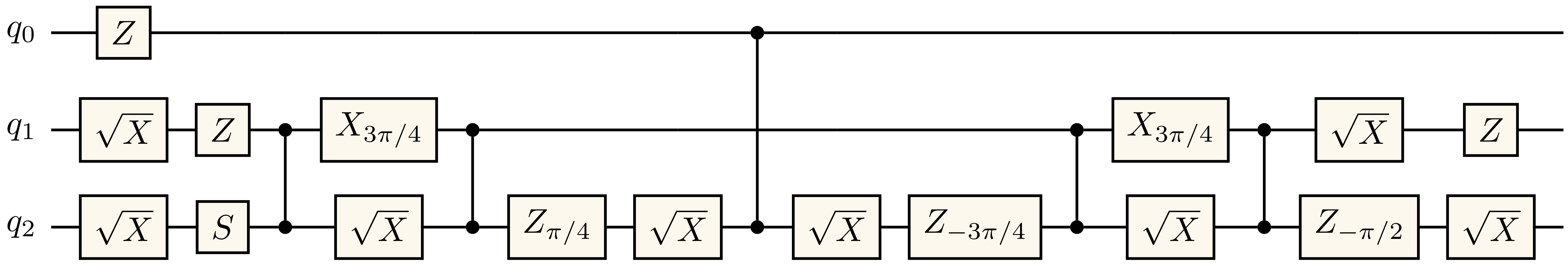

Quantum algorithms are sequences of gates acting on quantum states

<u>Compilation steps</u>

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- Map qubits in the algorithm to qubits on your hardware
- Insert SWAP gates to connect qubits far apart that need to interact
- Compress the resulting circuit

Summary

- Qubits can be in superposition states; exponentially many classical bits are required to describe many qubits
- Quantum algorithms are implemented by applying a sequence of single- and two-qubit gates (unitary matrices) to the qubits (states represented as vectors)
- Quantum algorithms need to be compiled to fit on the quantum hardware; the Solovay-Kitaev theorem tells us that universal gate sets can achieve this without prohibitive overhead to ensure precision